Exercise 41

How close to -3 do we have to take x so that

$$\frac{1}{(x+3)^4} > 10{,}000$$

Solution

Let δ be the distance from x to -3: $\delta = |x-(-3)| = |x+3|$. As a result,

$$\frac{1}{(x+3)^4} = \frac{1}{|x+3|^4} = \frac{1}{\delta^4}.$$

If $\delta < \frac{1}{10}$, then

$$\frac{1}{\delta^4} > \frac{1}{\frac{1}{10^4}} = 10\,000.$$

This upper bound of $1/10$ for δ is consistent with the graph of $1/(x+3)^4$.